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Route Switching in Cognitive Radio Networks

ABSTRACT

In Cognitive Radio Networks (CRNs), Secondary Users (SUs) are provided with the flexibility of accessing Primary Users' (PUs') idle spectrum bands but the availability of spectra is dynamic due to PUs' uncertain activities of channel reclamation. In the multi-hop CRNs consisting of SUs as relays, such spectrum mobility will cause the invalidity of pre-determined routes of data flows since some of the channels pre-assigned to the routes become unavailable. In this thesis, we investigate the spectrum-mobility-incurred route-switching problem in both spatial and frequency domain for CRNs, where the spatial switching determines which relays and links should be re-selected and the frequency switching decides which channels ought to be re-assigned to the spatial routes. We further formulate the route-switching problem as the *Route-Switching Game* which is shown to be a *potential game* and has a pure Nash Equilibrium (NE). Accordingly, efficient algorithms for finding the NE and the $\epsilon-$NE are proposed. Then we extend the proposed game to the incomplete-information scenario and provide a method to compute the Bayesian NE. Finally, we prove that the *price of anarchy* of the proposed game has an upper bound, and a virtual charging scheme is further introduced to help the obtained NE achieve the social optimality. The proposed route-switching scheme not only avoids conflicts with PUs but also mitigates spectrum congestion. Meanwhile, tradeoffs between *routing costs* and *channel switching costs* are achieved.

Keywords: Cognitive Radio Networks, Game Theory, Routing, Spectrum Mobility

认知无线电网络的路由切换机制

摘要

在认知无线电网络中，次级用户可以灵活地接入主用户的空闲频段。然而由于主用户会不定时地占用他们的法定许可信道，因此对于次级用户来说，其所能使用的频谱是不确定的并动态变化的。在由多个次级用户充当路由节点的多跳认知无线电网络中，这种频谱的不确定性将使得原有数据链路上已分配的信道变得不再可用，进而不可避免地造成网络中数据流的路由中断。在该毕业论文中，我们将从空间域和频域两个角度联合探究上述由频谱不确定性引发的路由中断切换问题。具体地，空间域的路由切换决定数据流应当重新选取网络空间中哪些节点和链路作为新的路径，而频域的路由切换则决定新路径上的链路应当重新分配哪些信道。我们进一步将上述问题阐述为“路由切换博弈”。该博弈被证明是一种势博弈，并且拥有一个纯策略纳什均衡点。接着，我们提出了几种有效的算法用以计算该纳什均衡和ϵ-纳什均衡点。继而我们有将博弈模型拓展到非完全信息模型下并给出了一个简单的算法计算贝叶斯-纳什均衡点。最后，我们证明了博弈的无秩序代价是确定的上界的并且进一步提出一种虚拟收费机制使得博弈均衡点同时实现社会最优性。我们提出的联合路由切换方案不仅避免了与主用户之间的频谱使用冲突，还有效地降低了频谱的拥塞。更重要的是，我们的方案实现了路由损耗和信道切换损耗之间的平衡。

关键词: 认知无线电网络，博弈论，路由，频谱不确定性

Contents

Chapter 1 Introduction

1.1 Cognitive Radio Network (CRN)

With the rapid progress of wireless communications technology, the demands for spectra have been going up at a tremendous speed in the past few years. For example, the development of Wireless Local Area Networks (WLAN) and Wireless Personal Area Networks (WPAN) lead to the fact that more and more people choose wireless access to the internet. Currently, the spectrum bands utilized by these technologies are mainly restricted within unlicensed bands. Unfortunately, the available spectrum resources in unlicensed bands are extremely limited, which will inevitably become saturated in the near future under the booming demands for wireless services (WLAN, WPAN, etc.). On the other hand, some types of wireless services (e.g., broadcast services and satellite services) impose high demands on the communication quality and need protections against potential interference incurred by other services. In order to provide good protections, frequency regulators like Federal Communications Commission (FCC) choose to assign exclusive spectrum bands (licensed bands) to these service providers. In fact, the amount of licensed spectra greatly exceeds unlicensed ones since most of our spectrum resources are used as licensed bands. However, in contrast to the large amount of licensed spectra, its spectrum utilization is very low, which causes the following undesirable fact: a small portion of spectra (unlicensed bands) support the majority of wireless services while a large part of spectra (licensed bands) are heavily underutilized. Therefore, we can reach the conclusion that most of our spectrum resources are wasted under the current licensed/unlicensed spectrum allocation scheme.

In face of the above dilemma, the Cognitive Radio was proposed as a promising paradigm for relieving the spectrum scarcity. Generally speaking, the cognitive radio, established on the software-defined radios, is defined as a kind of intelligent wireless communication system which is aware of its surrounding and uses the methods of “understanding-by-building” to learn from the surroundings and adapt to variations of the input statistical data ([1], Mitola, 2006: 1-30.). With the ability to sense the communication environment, cognitive radios can actively find the idle spectrum opportunities (spectrum holes) and automatically tuned to the idle channel, without the interference with spectrum holders. As a result, cognitive radios efficiently improves the spectrum utilization and adapts to the flexible communication needs while conforming to the rules of spectrum regulators.

1.2 Spectrum Mobility in Cognitive Radio Networks

As is mentioned above, in Cognitive Radio Networks (CRNs), unlicensed users or Secondary Users (SUs) who require the usage of spectra can actively sense the states of licensed bands. If these bands are not occupied (or occasionally occupied) by licensed users or Primary Users (PUs), SUs are allowed to access the idle licensed spectra under appropriate negotiations with PUs. Such a method is also referred as Dynamic Spectrum Access (DSA).

From the perspective of network designers, DSA provides SUs with high flexibility in selecting spectra but brings new challenges to the design of CRNs at the same time, one of which is the *spectrum mobility*.

In CRNs, PUs can reclaim their licensed channels at any time due to their high priority of channel occupation, and SUs must cease their transmission [1] on those spectrum bands. Hence, from SUs' perspective, the availability of spectra is dynamic due to PUs' uncertain activities of channel reclamation, which causes *spectrum mobility* in CRNs.

Spectrum mobility is one of the most important features of cognitive radio networks, whose existence impose lots of challenges on network design. For example, spectrum mobility further increases the channel heterogeneity perceived by SUs, thus causing the difficulties in allocating spectra in the secondary network ([3], Wang et al., 2010: 1.), ([4], Gao et al., 2011: 1.). Moreover, proper spectrum switching schemes must also be investigated in order to ensure the smooth channel transition and the desirable

[1] Such a method is also referred as the spectrum overlay mode. An alternative way is to insure that the amount of generated interference to PUs is below a certain threshold, namely the spectrum underlay mode ([2], Southwell et al., 2012: 1.). In this thesis, we only consider the overlay mode.

communication quality for SUs ([2], Southwell et al., 2012: 1.), ([5], Liang et al., 2012: 2.).

In this thesis, we will put our attention on the influences of spectrum mobility in multi-hop cognitive radio networks, which is demonstrated in the following section.

1.3 Two-Dimensional Route Switching in Cognitive Radio Networks

In the context of multi-hop CRNs where multiple SUs act as potential relays[2], one of the major influences of spectrum mobility is the break of routes of incoming data flows since the unavailability of PUs' channels disables the transmission over some links on the pre-determined routes. To avoid conflicts with PUs and resume routing, each flow initiator can either inform intermediate SU relays to switch their accessing channels or re-select a new spatial route[3] where channels are not reclaimed. However, the following tradeoff implies that the two-dimensional route switching (i.e., the combination of both channel switching and spatial route re-selection) is a better choice.

On one hand, by maintaining previous (and possibly the best) spatial routes and choosing less congested (and idle) channels as the targets, channel switching can efficiently avoid conflicts with PUs and reduce *routing costs* incurred by relaying data, including the transmission delay, energy consumption, etc. Unfortunately, frequent channel switching could also cause significant *switching costs* such as the additional power consumption and switching delay. On the other hand, re-selecting a new spatial route can yield fewer *switching costs* but may lead to additional *routing costs* at the same time. Consequently, there's a *tradeoff* between the two costs, which must be achieved by switching routes in both spatial and frequency domain.

Additionally, it should be mentioned that the above two-dimensional route switching is different from the traditional joint design of route selection and channel allocation. First, the former problem is unique to CRNs while the latter one is intended for general wireless networks which cannot capture the traits of CRNs like the spectrum

[2]Since we focus on routing in the secondary network, we will use "multi-hop CRN" and "multi-hop secondary network" interchangeably in this thesis.

[3]In the following, we will refer the selection of intermediate nodes and edges as *spatial routes* and the choice of channels exploited on the spatial routes as *frequency routes*.

mobility. Most importantly, the former problem is "history-relevant" while the latter one is usually "history-free". In the route-switching problem, any decisions like channel switching are highly related to the routing history such as pre-determined routes and channel allocation, and must weigh the benefits and costs of violating previous choices. The tradeoff mentioned above is one typical example. By comparison, neither spatial nor frequency route history exists in the context of traditional joint design problems. In fact, the latter problem can be seen as a special case of the former one when the routing history is zero.

In this thesis, we propose Route-Switching Games to address the above *spectrum-mobility-incurred route-switching* problem in CRNs. The major contributions of this thesis lie in the following aspects.

• To our best knowledge, this thesis is the first to investigate the spectrum-mobility-incurred route-switching problem in CRNs. Our scheme not only avoids the conflicts with PUs but also mitigates the spectrum congestion and achieves the tradeoff between routing costs and channel switching costs.

• We formulate the proposed problem as the Route-Switching Game which is proved be a *potential game*. Efficient algorithms for finding the Nash Equilibrium (NE) and the $\epsilon-$NE are provided in this thesis.

• We further study the game with incomplete information, where players' parameters are private. In such a scenario, a Bayesian NE is proved to exist and an algorithm for calculating the Bayesian NE is offered.

• We compare the performance of the NE (and the Bayesian NE) in the proposed game with the socially optimal results in terms of the social costs (and the expected social costs), i.e., the *Price of Anarchy* (and the *Bayesian Price of Anarchy*), which is upper-bounded by deterministic factors.

• To improve the efficiency of the NE, we further introduce a charging scheme to influence players' strategies, which helps the proposed game achieve the minimum social costs as centralized schemes do.

The rest parts of this thesis is organized as the following. We will first introduce the system model in chapter 2. Next, in chapter 3 and 4, Route-Switching Games with complete and incomplete information will be demonstrated, respectively. Then we will analyze the price of anarchy in chapter 5 and provide a charging scheme to improve the performance of the proposed game in chapter 6. Finally, simulation results, related works and conclusions will be given in chapters 7, 8 and 9, respectively.

Chapter 2 Network Model

2.1 Architecture of Multi-hop CRNs

We consider a multi-hop CRN where multiple SUs act as routers for the incoming data flows, and there're C orthogonal channels (with the same bandwidth) that can be accessed by SUs when they are not occupied by PUs (each channel is denoted by $j \in \mathcal{C} = \{1, 2, \cdots, C\}$). For the simplicity of analysis, we assume the entire secondary network lies in the same "collision domain" with PUs[1], i.e., the perceived channel states (either busy or idle) at each SU are identical in the considered network. This assumption is valid for many geographically-centric secondary networks coexisting with powerful PU transceivers, like PU base stations in cellular networks, as is shown in Figure 2.1.

Formally, the entire secondary network can be characterized by a topological graph $G = (V, E)$. Here, V is the set of nodes (SUs) and E is the set of edges in the topological graph. Note that there's an edge $E_{u,v}$ between a pair of nodes (u, v) iff they're within the transmission range of each other, so an edge corresponds with a data link. However, for a link to be able to transmit data, it must be allocated a traffic channel. As our focus is the *route-switching* problem, we suppose there have been pre-determined channel allocations on each link (but these pre-assigned channels may be reclaimed by PUs and become unavailable now). Here, we denote matrix A the indication of pre-assigned channels on different links. Specifically, its element $A_{e,j} = 1$ implies that channel j was pre-assigned to link e and $A_{e,j} = 0$ otherwise. Hence, matrix A represents a part of the "routing history", and such history is local information maintained at each intermediate SU router.

[1] Note that our scheme can also be modified to incorporate the spatial diversity of PUs' spectra in secondary networks.

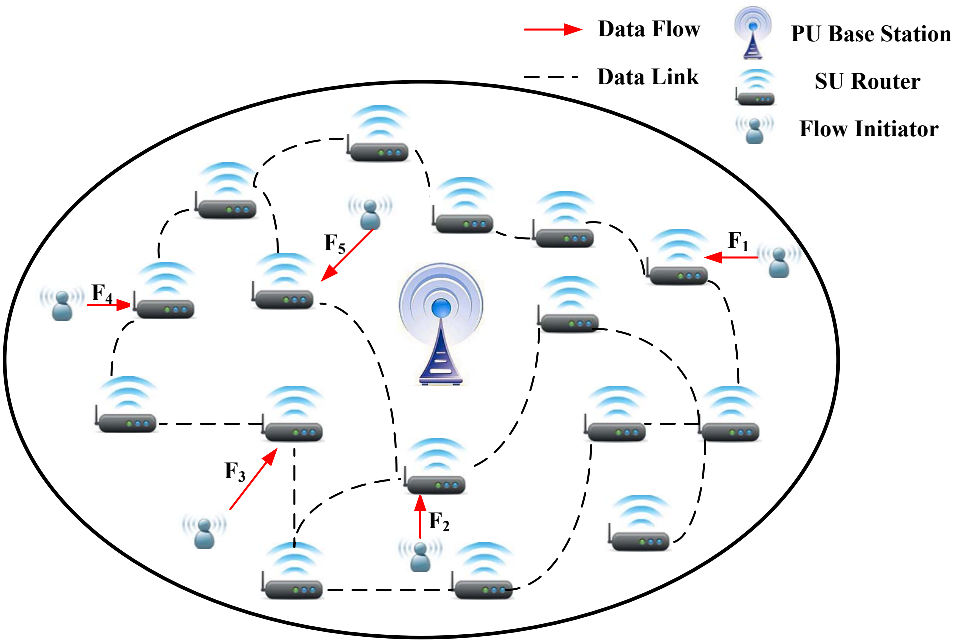

Figure 2.1 An example of the multi-hop and multi-flow CRN. Note that the entire secondary network is within the transmission range of the PU base station, so the spectrum opportunities perceived at each SU are identical.

2.2 Flow Model

Suppose there're M concurrent and constant[2] data flow inputs (denoted by $F_k,\ k \in \mathcal{M} = \{1, 2, \cdots, M\}$) which need routing among the secondary network, and denote the source and destination of data flow F_k by a pair (S_k, D_k). For the efficiency and reliability of flow transmission, F_k $(k \in \mathcal{M})$ segments its data by packets, each with size μ_k. Additionally, we denote the flow rate of F_k by r_k and assume that those data flows are from different initiators, each hoping to minimize its own costs incurred in the routing process.

2.3 Spectrum Mobility and Route Switching

When high-priority PUs reclaim their licensed channels, SUs must cease their transmission on those spectrum bands, which causes the *spectrum mobility*. Here, we denote Γ the set of all currently *unavailable* channels due to PUs' reclamation, which can be

[2]We assume those data flows can last for a period of time like minutes or hours, which is particularly suitable for characterizing multimedia streaming, P2P downloading, etc.

obtained by flow initiators without incurring significant overhead costs through our implementation (see section 2.6).

Unlike many previous works proposing statistical models to characterize PUs' reclaiming activities ([2], Southwell et al., 2012: 3.), ([6], Yarkan and Arslan, 2007: 2.), we do not predict PUs' behaviors, i.e., our scheme is *reactive*, since the precision of predictions still remains a major problem. Besides, route-switching schemes should provide routing reliability as much as possible, instead of probabilistic results, because the focus of the proposed problem is exactly to handle the negatives effects of PUs' spectrum uncertainty, which is the other reason why we don't choose proactive models.

In face of spectrum mobility, routes must be switched in both spatial and frequency domain so as to avoid the conflicts with PUs, mitigate the congestion, and balance the routing costs and channel switching costs (see section 2.5 for the formal definition), since some pre-assigned channels over certain links become invalid, which is equivalent to the break of routes. Here, we use a two-dimensional vector $\mathbf{X}$ to characterize the *newly selected* spatial and frequency routes in CRNs, which is also the strategy variable in the considered problem. Specifically, its element $X_{e,j}^{k} = 1$ when link e is included in the new spatial route of data flow F_k and channel j is re-selected for this link ($X_{e,j}^{k}$=0 otherwise).

2.4 Interference Model and Constraints

We use the protocol interference model ([7], Gupta and Kumar, 2000: 3-4.), where the transmission in channel j over link e succeeds if all potential interferers in the *interference neighborhood* of link e remain silent in channel j for the transmission duration. Here, the *interference neighborhood* of link e, i.e., $I(e)$, is the set of links whose end nodes have interference links or data links incident on the end nodes of e. Further, when channel j is perceived idle over link e, the contention window is activated and link e will contend for the transmission opportunities with all interfering links in $I(e)$ (specifically, it's the transmitter on one end of link e that executes the contention). This model resembles CSMA/CA in IEEE 802.11, based on an RTS-CTS-Data-ACK sequence.

Moreover, there're several constraints that should be considered. Though these constraints *do not* influence the following theoretical analysis, we impose them to cater to the current hardware development and make our results more meaningful. Later in section 3.3, we will further reflect them in the computation of the NE.

- **Constraint 1:**

$$\sum_{j \in \mathcal{C}} X_{e,j}^{k} \leq 1, \ \forall k \in \mathcal{M}, e \in E. \tag{2-1}$$

This constraint implies that a certain flow can use at most one channel over a certain link. We make this constraint because some routers cannot execute efficient packet segmentation and recovery to support simultaneous transmission of a single packet in multiple channels.

- **Constraint 2:**

$$\sum_{k \in \mathcal{M}} X_{e,j}^{k} \leq 1, \ \forall e \in E, j \in \mathcal{C}. \tag{2-2}$$

This constraint indicates a certain channel can only be occupied by at most one flow over a certain link, considering the significant co-channel collisions and interference on the same link.

- **Constraint 3:**

$$\sum_{e \in E(v)} \sum_{k \in \mathcal{M}} \sum_{j \in \mathcal{C}} X_{e,j}^{k} \leq \alpha_v, \ \forall v \in V, \tag{2-3}$$

where $E(v)$ is the set of edges associated with node v and α_v is number of radios that node v equips. This constraint shows that the number of channels associated with a certain SU should not exceed its radio limitation.

2.5 Cost Model

We mainly consider two types of costs in our model: routing costs and switching costs, which will be further discussed in the rest of this section. Additionally, it should be mentioned that the following costs are derived under the premise that channel $j \in \mathcal{C}$ is sensed to be *idle*, i.e., $j \notin \Gamma$.

2.5.1 Routing Cost

Routing costs characterize the potential expense incurred by relaying packets on established routes, including the routing delay, power consumption, etc. Here, we concentrate on the following two major routing costs.

• *Delay Cost:* Under the interference model mentioned above, significant transmission delay will be incurred if a channel is congested, since SUs must contend and wait for transmission opportunities. By comparison, other minor delay (e.g., propagation delay) is neglected in this thesis.

As is typical of many random access protocols ([8], Abramson, 1970: 1-5.), ([9], Stallings, 2005: 345-380.), we make the following assumption: in any contention window and any channel $j \in \mathcal{C}$, a certain link $e \in E$ and all its interfering links have the same probability of wining the access to this channel. Following the methods provided in ([10], Rappaport, 2002: 247-250.), we can calculate the *expected* transmission delay under our interference model, which characterizes the expected waiting time before a certain link e wins the opportunity to transmit *one packet* in channel j:

$$\lambda_{e,j} = \sum_{e' \in I(e)} \sum_{k \in \mathcal{M}} X_{e',j}^{k} \omega_k, \tag{2-4}$$

where $\omega_k := \frac{\mu_k}{r_k}$ is the transmission time required by flow F_k for one packet. The derivation of equation (2-4) is beyond the scope of this thesis, so we omit it for brevity. The intuition behind equation (2-4) is explained as the following. $\sum_{k \in \mathcal{M}} X_{e',j}^{k} \omega_k$ in equation (2-4) represents the traffic demands (for transmission time) in channel j over link e' imposed by all passing data flows, and thus equation (2-4) is the aggregate traffic demands in channel j from the interference neighborhood of link e. Generally speaking, $\lambda_{e,j}$ reflects the *congestion level* of channel j perceived over link e, and delay costs can also be interpreted as the *congestion costs* in our model.

For the denoting simplicity, we introduce a 0-1 indicator $\theta_{e,e'}$ to imply the interference relationship. Specifically, $\theta_{e,e'} = 1$ means that link e' is in the interference neighborhood of e and $\theta_{e,e'} = 0$ otherwise. Note that $\theta_{e,e} = 0$ $(\forall e \in E)$ and we consider the mutual interference, so $\theta_{e,e'} = \theta_{e',e}$. Besides, the congestion caused by one's own transmission over other interfering links is neglected for the tractability of analysis, since recent literatures ([11], Choi and Park, 2012: 1-2.), ([12], Chun et al., 2009: 1-7.) have suggested such congestion can be mitigated significantly by exploiting the self-interference cancellation technology in relay systems. Therefore, we can rewrite the expected delay perceived by F_k when it is transmitted in channel j over link e by:

$$\lambda_{e,j}^{k} = \sum_{e' \in E} \sum_{k' \in \mathcal{M}_k} X_{e',j}^{k'} \omega_{k'} \theta_{e,e'}, \tag{2-5}$$

where $\mathcal{M}_k = \mathcal{M} \backslash \{k\}$ denotes set $\mathcal{M}$ excluding set $\{k\}$. Further, the total expected delay incurred on flow F_k's two-dimensional route is:

$$DC_k = \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^{k} \lambda_{e,j}^{k}. \tag{2-6}$$

In the rest of this thesis, we will use DC_k to characterize the delay costs for flow F_k.

- *Energy Cost:* We mainly consider the energy used for data transmission. Under our interference model, when one SU transmits in channel j over link e, other SUs within $I(e)$ must remain silent in channel j, so the SINR perceived at each SU receiver is merely dependent on the intrinsic channel quality and geographical conditions, such as the AWGN, path loss, etc. Further, according to Shannon Formula, energy costs incurred by transmitting per packet of F_k's data in channel j over link e are only related to the flow rate, packet size (which influences the transmission duration), channel quality and geographical conditions. We model such energy costs as a general form $\varphi_{e,j}(r_k, \mu_k)$ and $\varphi_{e,j}^{k}$ for short. Then, the total energy costs on F_k's route are

$$EC_k = \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^{k} \varphi_{e,j}^{k}. \tag{2-7}$$

2.5.2 Switching Cost

The other kind of costs are caused by channel switching. Each time switching happens at intermediate SU routers, switching costs are incurred, including additional energy consumption used for establishing new connections, additional costs of sensing, switching delay, etc. Here, we use γ to indicate the overall costs per time of channel switching. Note that γ includes the costs of both ON→OFF switching (tearing down the old channel connection) and OFF→ON switching (establishing the new channel connection), so the two types of switching costs can be seen as being incurred altogether in *either* switching scenario. In this thesis, we assume the overall switching costs γ are incurred only in the OFF→ON transition. Therefore, the total channel switching costs caused by F_k's re-selection strategy are

$$SC_k = \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^k (1 - A_{e,j}) \gamma. \tag{2-8}$$

Corresponding to the above discussion, equation (2-8) implies that only when $A_{e,j} = 0$ and $X_{e,j}^k = 1$ (i.e., link e did not use channel j previously, but now this channel is allocated to e according to F_k's strategy), switching costs are incurred.

As is stated in the introduction, switching costs need to be balanced with routing costs, so we model flow F_k's *total costs* as:

$$TC_k = DC_k + EC_k + SC_k. \tag{2-9}$$

Additionally, to facilitate the reading, we list the major notations in Table 2.1.

2.6 Implementation

We briefly discuss the implementation of the proposed game. There would be a set of separate control channels, different from those owned by PUs. Before the flow transmission, nodes (i.e., SUs) will first perform spectrum sensing to obtain the states of

Table 2.1 Major Notations

$\mathbf{A}$	matrix notation of the channel assignment history
$A_{e,j}$	the 0-1 indicator of whether channel j was previously assigned to link e
$\mathbf{X}$	matrix notation of the newly-selected routes and channels
$X^k_{e,j}$	the 0-1 indicator of whether link e is included in flow F_k's new routes and channel j is assigned to this link
s	the strategy profile (an alternate notation of $\mathbf{X}$)
s_k	equals to $\{(e,j)\|e \in E, j \in \mathcal{C}, X^k_{e,j} = 1\}$, denoting flow F_k's new joint selection of routes and channels
γ	the switching costs per time of channel handoff
$\lambda^k_{e,j}$	the delay costs of flow F_k incurred by the transmission in channel j over link e
$\varphi^k_{e,j}$	the energy costs of flow F_k incurred by the transmission in channel j over link e
μ_k	the packet size of data flow F_k
r_k	the flow rate of data flow F_k
ω_k	equals to μ_k/r_k, denoting the required time for transmitting one of flow F_k's packet
$I(e)$	the interference neighborhood of link e
$\theta_{e,e'}$	the 0-1 indicator of whether link e interferes with link e'
$\Phi(s)$	the potential function under strategy profile s

channels[3]. If the available channels cannot sustain the existing routes, nodes where the routes break will broadcast route-switching messages to flow initiators through the control channels and the flooding scheme. Each route-switching message contains the affected flows' indices as well as the indices of channels whose state changes so that each initiator can obtain Γ (i.e., the set of unavailable channels). Then initiators will distributively play the Route-Switching Game to re-select the two-dimensional routes and inform the intermediate nodes. Here, initiators can either include the new routes in the header of packets like in DSR or inform nodes through the control channels. Finally, new routes are built.

In terms of the overheads, only messages from a small fraction of nodes (i.e., n-odes where the routes break) and the states of a part of channels (i.e., channels whose state changes) are communicated in the network, which implies that the overheads are relatively low. Besides, we consider the case where the topological information (i.e., $G(V,E)$) and intrinsic channel conditions (i.e., AWGN, path loss, etc.) change in a timescale (e.g., several hours) much longer than the routing period (e.g., several sec-

[3]To reduce the sensing overheads, efficient sensing schemes like cooperative sensing ([13], Yucek and Arslan, 2009: 9-10.), compressive sensing ([14], Fanzi et al., 2011: 1-12.), etc, can be adopted here.

onds or minutes), so it can be measured and reported by nodes in advance, and stored in the database of flow initiators beforehand, without incurring additional overheads each time two-dimensional routes are *switched*.

Chapter 3 Route-Switching Games with Complete Information

Equation (2-5) indicates a certain flow F_k's delay costs incurred in channel $j \in \mathcal{C}$ over link $e \in E$ are also dependent on other flow's route-switching strategies (i.e., the selection of $X^{k'}_{e',j}$, $\forall k' \in \mathcal{M}_k$, $e' \in I(e)$), so we formulate the above problem as Routing-Switching Games, where players (i.e., **flow initiators**) distributively and selfishly *switch* their two-dimensional routes in face of spectrum mobility, aiming at minimizing their *total costs*.

3.1 Game Formulation

In the complete-information scenario, each player's (flow's) information (i.e., data rate r_k and packet size μ_k, $\forall k \in \mathcal{M}$) is known to others. Here, we can use a tuple $\mathcal{G} = \{G, \mathbf{A}, \Gamma, r, \mu, TC, S\}$ to denote the Route-Switching Game with complete information. Here, the meanings of G, $\mathbf{A}$ and Γ have been explained in section 2. $r = \{r_1, \cdots, r_M\}$ and $\mu = \{\mu_1, \cdots, \mu_M\}$ are publicly-known parameter vectors of flows. TC is the set of players' cost functions, shown in (2-9). $S = \{(e,j)|e \in E, j \in \mathcal{C}\}$ is the two-dimensional strategy space. In this thesis, we consider the *symmetric game* where all players have the same strategy space. Further, we denote $s = \{s_1, \cdots, s_M\}$ the strategy profile, where $s_k = \{(e,j) \in S|X^k_{e,j} = 1\}$ is F_k's route-repick strategy[1]. S-ince F_k's costs and $\lambda^k_{e,j}$ are relevant to the strategy profile s, we denote them by $DC_k(s)$, $EC_k(s)$, $SC_k(s)$, $TC_k(s)$ and $\lambda^k_{e,j}(s)$, respectively.

In addition, it's worthy mentioning that the above formulation does not impose any constraints on the connectivity of switched routes but such an omission won't influence any of the following analytical results. Instead, we guarantee the connectivity through our algorithm implementation (see Theorem 3 in section 3.3).

Finally in this section, we give the definition of the Nash Equilibrium[2], which will

[1] In this thesis, each player F_k's ($\forall k \in \mathcal{M}$) strategy can be expressed in two forms: s_k and the corresponding 0-1 indication $X^k_{e,j}$ ($e \in E, j \in \mathcal{C}$). The two forms are equivalent and will be used interchangeably in the following. Note that $\sum_{e\in E}\sum_{j\in\mathcal{C}} X^k_{e,j}$ is the same as $\sum_{(e,j)\in s_k}$.

[2] We only consider the pure NE throughout this thesis.

be frequently discussed.

Definition 1 (Nash Equilibrium): A strategy profile $s^* = (s_1^*, s_2^*, \cdots, s_M^*)$ is a Nash Equilibrium if for any player F_k ($\forall k \in \mathcal{M}$) and its any strategy $s_k \subseteq S$,

$$TC_k(s_k^*; s_{-k}^*) \leq TC_k(s_k; s_{-k}^*),$$

where s_{-k}^* is the strategy profile s^* excluding s_k^*. By definition, no player can reduce its own costs by unilaterally changing the strategy at the equilibrium.

3.2 Potential Game

The potential game ([15], Mondere and Shapley, 1996: 1-20.) is a relatively new game-theoretical model which can characterize a wide range of games, including the classical congestion game ([16], Christodoulou and Koutsoupias, 2005: 1-7.). It has already demonstrated its importance through many successful applications to practical problems like spatial spectrum access ([17], Chen and Huang, 2012: 1-10.), ([18], Nie and Comaniciu, 2005: 1-10.), gateway selections ([19], Song et al., 2011: 1-12.), etc.

In the rest of this section, we will briefly introduce the concept of the potential game and its properties, which will be further exploited in this thesis.

Definition 2 (Potential Game): A game is referred as the potential game if and only if there exists a potential function in the game.

Definition 3 (Potential Function): A function $\Phi(s)$ is the potential function for the minimum game[3] $\mathcal{G}$ if for any strategy profile s, any player F_k ($\forall k \in \mathcal{M}$) and its any two strategies $s_k, s_k^{'} \subseteq S$

$$TC_k(s_k^{'}; s_{-k}) - TC_k(s_k; s_{-k}) < 0$$
$$\Rightarrow \Phi(s_k^{'}; s_{-k}) - \Phi(s_k; s_{-k}) < 0.$$

Potential games have many ideal properties, and we mainly use three of them in this thesis.

[3] A game is a *minimum game* if players tend to *minimize* their cost functions.

Property 1: Every finite potential game[4] has at least one pure Nash Equilibrium.

From the definition of the potential function, we can observe that the minimum of the potential function corresponds with a pure NE in the minimum game since at the minimum, no player can unilaterally decrease its own costs otherwise the decrease in this player's costs will also lead to the reduction in the potential function, violating the definition of the minimum. Conversely, any pure NE in a potential game is also a minimum of the potential function, otherwise there exists at least one player who can decrease its own costs by reducing the potential function towards the minimum, which deviates from the definition of NE.

Property 2: Every finite potential game has the *Finite Improvement Property* (FIP).

The meaning of FIP is as the following. Initially, each player can randomly select its own strategy. Then every player rotates to improve its strategy by reducing the potential function at each step with others' strategies fixed. After finite improvement steps, the potential function will reach the minimum, and thus an NE is derived. FIP actually provides us with a feasible method to compute an NE in the potential game, which will be further exploited in section 3.3.

Property 3: Every potential game has at least one pure ϵ-Nash Equilibrium.

We won't explain the details of Property 3 here. Further discussions will be given in section 3.4.

Proofs to the three properties can be found in ([15], Mondere and Shapley, 1996: 4-10.).

3.3 Existence and Computation of the NE

In this section, we will first prove that the Route-Switching Game is essentially a potential game. Then an algorithm for computing the NE is provided.

Theorem 1: Under complete information, the Route-Switching Game is a finite

[4] A game is said to be finite when each player has a finite number of options and the number of players is also finite.

potential game which has the potential function:

$$\Phi(s) = \sum_{k \in \mathcal{M}} \omega_k [DC_k(s) + 2EC_k(s) + 2SC_k(s)]. \tag{3-1}$$

Proof. It's obvious that the Route-Switching Game is finite, and we only prove that (3-1) is the potential function of the proposed game. Consider an *improvement* from strategy profile s to q. The 0-1 strategy indication is denoted by $X^k_{e,j}$ for s and $X'^k_{e,j}$ for q, $\forall k \in \mathcal{M}, e \in E, j \in \mathcal{C}$. The only difference between s and q is that player F_k *improves* its strategy from s_k to q_k, i.e., $s \backslash \{s_k\} = q \backslash \{q_k\}$. Then we have:

$$TC_k(s) > TC_k(q),$$

which means that

$$\begin{aligned} &\sum_{e \in E} \sum_{j \in \mathcal{C}} X^k_{e,j} [\lambda^k_{e,j}(s) + \varphi^k_{e,j} + (1 - A_{e,j})\gamma] \\ > &\sum_{e \in E} \sum_{j \in \mathcal{C}} X'^k_{e,j} [\lambda^k_{e,j}(q) + \varphi^k_{e,j} + (1 - A_{e,j})\gamma]. \end{aligned}$$

At the same time, we define:

$$\zeta^k_{e,j}(s) := \omega_k [\lambda^k_{e,j}(s) + 2\varphi^k_{e,j} + 2(1 - A_{e,j})\gamma].$$

Thus we can derive

$$\begin{aligned} &\Phi(s) - \Phi(q) \\ &= \sum_{k' \in \mathcal{M}} \sum_{(e,j) \in s_{k'}} \zeta^{k'}_{e,j}(s) - \sum_{k' \in \mathcal{M}} \sum_{(e,j) \in q_{k'}} \zeta^{k'}_{e,j}(q) \\ &= [\sum_{(e,j) \in s_k} \zeta^k_{e,j}(s) - \sum_{(e,j) \in q_k} \zeta^k_{e,j}(q)] \\ &+ [\sum_{k' \in \mathcal{M}_k} \sum_{(e,j) \in s_{k'}} \zeta^{k'}_{e,j}(s) - \sum_{k' \in \mathcal{M}_k} \sum_{(e,j) \in q_{k'}} \zeta^{k'}_{e,j}(q)]. \end{aligned} \tag{3-2}$$

For the first term in the above equation,

$$\begin{aligned}&\sum_{(e,j)\in s_k}\zeta^k_{e,j}(s)-\sum_{(e,j)\in q_k}\zeta^k_{e,j}(q)\\&=\sum_{e\in E}\sum_{j\in\mathcal{C}}X^k_{e,j}\omega_k[\lambda^k_{e,j}(s)+2\varphi^k_{e,j}+2(1-A_{e,j})\gamma]\\&\quad-\sum_{e\in E}\sum_{j\in\mathcal{C}}X'^k_{e,j}\omega_k[\lambda^k_{e,j}(q)+2\varphi^k_{e,j}+2(1-A_{e,j})\gamma].\end{aligned}\tag{3-3}$$

For the second term in (3-2), we should first notice that

$$s_{k'}=q_{k'},\ \forall k'\in\mathcal{M}_k,$$

i.e.,

$$X^{k'}_{e,j}=X'^{k'}_{e,j},\ \forall k'\in\mathcal{M}_k, e\in E, j\in\mathcal{C}.\tag{3-4}$$

Then the second term can be equally written as

$$\begin{aligned}&\sum_{k'\in\mathcal{M}_k}\sum_{e\in E}\sum_{j\in\mathcal{C}}X^{k'}_{e,j}\zeta^{k'}_{e,j}(s)-\sum_{k'\in\mathcal{M}_k}\sum_{e\in E}\sum_{j\in\mathcal{C}}X'^{k'}_{e,j}\zeta^{k'}_{e,j}(q)\\&=\sum_{k'\in\mathcal{M}_k}\sum_{e\in E}\sum_{j\in\mathcal{C}}X^{k'}_{e,j}[\zeta^{k'}_{e,j}(s)-\zeta^{k'}_{e,j}(q)]\\&=\sum_{k'\in\mathcal{M}_k}\sum_{e\in E}\sum_{j\in\mathcal{C}}X^{k'}_{e,j}\omega_{k'}[\lambda^{k'}_{e,j}(s)-\lambda^{k'}_{e,j}(q)]\\&=\sum_{k'\in\mathcal{M}_k}\sum_{j\in\mathcal{C}}\sum_{e\in E}\sum_{e'\in E}X^{k'}_{e,j}\omega_{k'}\omega_k\theta_{e,e'}(X^k_{e',j}-X'^k_{e',j}).\end{aligned}\tag{3-5}$$

Note that the deduction of (3-5) exploits the fact that

$$\lambda^{k'}_{e,j}(s)-\lambda^{k'}_{e,j}(q)=\sum_{e'\in E}\omega_k\theta_{e,e'}(X^k_{e',j}-X'^k_{e',j}),$$
$$\forall k'\in\mathcal{M}_k, e\in E, j\in\mathcal{C},$$

since only F_k's strategy changes while others' routes remain the same.

Interchange the role of e and e' together with (3-4) and the assumption $\theta_{e,e'}=\theta_{e',e}$, equation (3-5) can also be written as

$$\begin{aligned}
&\sum_{e\in E}\sum_{j\in\mathcal{C}} X^{k}_{e,j}\omega_k \sum_{k'\in\mathcal{M}_k}\sum_{e'\in E} X^{k'}_{e',j}\omega_{k'}\theta_{e,e'} \\
&-\sum_{e\in E}\sum_{j\in\mathcal{C}} X'^{k}_{e,j}\omega_k \sum_{k'\in\mathcal{M}_k}\sum_{e'\in E} X'^{k'}_{e',j}\omega_{k'}\theta_{e,e'} \\
=&\sum_{e\in E}\sum_{j\in\mathcal{C}} X^{k}_{e,j}\omega_k\lambda^{k}_{e,j}(s) - \sum_{e\in E}\sum_{j\in\mathcal{C}} X'^{k}_{e,j}\omega_k\lambda^{k}_{e,j}(q).
\end{aligned} \tag{3-6}$$

By summing (3-3) and (3-6), we finally obtain that

$$\begin{aligned}
&\Phi(s) - \Phi(q) \\
=&\sum_{e\in E}\sum_{j\in\mathcal{C}} X^{k}_{e,j}\omega_k[2\lambda^{k}_{e,j}(s) + 2\varphi^{k}_{e,j} + 2(1-A_{e,j})\gamma] \\
&-\sum_{e\in E}\sum_{j\in\mathcal{C}} X'^{k}_{e,j}\omega_k[2\lambda^{k}_{e,j}(q) + 2\varphi^{k}_{e,j} + 2(1-A_{e,j})\gamma] \\
=&2\omega_k[TC_k(s) - TC_k(q)] > 0.
\end{aligned} \tag{3-7}$$

Hence, we have proved that (3-1) is the potential function of the proposed game. □

Theorem 2: There's a unique Nash Equilibrium in the Route-Switching Game with complete information.

Proof. The existence of the pure NE directly follows Property 1 of potential games, which we won't formally discuss here. As for the uniqueness, note that $\Phi(s)$ is actually a linear function in a close hyperplane, so it only has a single minimum point. As is mentioned in section 3.2, every pure NE in a potential game must also be a minimum of the potential function, so the Route-Switching Game (with complete information) only has one unique NE. □

Next, we will design an algorithm for finding the NE in the Route-Switching Game, shown in **Algorithm 1**. This algorithm is actually an iterative algorithm following FIP. Its major part is the strategy improvement (or update), which is done by first converting the reduction of the potential function to finding the shortest path in an undirected graph and then applying the well-known *Dijsktra Algorithm* to find such a path. Step

8 in Algorithm 1 indicates channels reclaimed by PUs cannot be exploited by SUs anymore. Step 9 and 10 handle the constraints mentioned in section 2.4, where Ω is the set of "(link, channel)" pairs that will deviate Constraint 2 if they are included in F_k's two-dimensional route, and Λ is the set of nodes that violate Constraint 3. Note that Constraint 1 has already been satisfied through the implementation of Dijsktra Algorithm since at most one edge is chosen between a certain pair of nodes. m in the algorithm acts like a counter recording the *subsequent* times for which players cannot reduce the value of the potential function, and the stop condition (step 19) indicates that all M players cannot reduce the potential function anymore, where the minimum point (NE) is reached. Note that a player's strategy is updated only when it can reduce the potential function (step 13 and 14) otherwise its previous strategy remains. Besides, the expression of $W_{(e,j)}$ in step 6 when F_k is improving its strategy is given by

$$\begin{aligned} W_{(e,j)} = & \sum_{k' \in \mathcal{M}_k} \sum_{e' \in E} X_{e',j}^{k'} 2\omega_k \omega_{k'} \theta_{e,e'} \\ & + 2\omega_k \varphi_{e,j}^k + 2\omega_k (1 - A_{e,j})\gamma, \end{aligned} \tag{3-8}$$

The correctness of (3-8) and Algorithm 1 is shown in the proof to Theorem 3.

Theorem 3: Each improvement step in Algorithm 1 can reduce the potential function to the maximum extent and guarantee the route connectivity in polynomial time with time complexity $O(|E||\mathcal{C}| + |V|^2)$.

Proof. Suppose F_k is updating its strategy. Then we have

$$\Phi(s) = \sum_{k' \in \mathcal{M}_k} \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^{k'} \zeta_{e,j}^{k'}(s) + \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^{k} \zeta_{e,j}^{k}(s),$$

where

$$\zeta_{e,j}^k(s) := \omega_k [\lambda_{e,j}^k(s) + 2\varphi_{e,j}^k + 2(1 - A_{e,j})\gamma].$$

Since $X_{e,j}^{k'}$ ($\forall k' \in \mathcal{M}_k, e \in E, j \in \mathcal{C}$) has been fixed when F_k is updating its strategy,

Algorithm 1 Find the Nash Equilibrium s^* in the Route-Switching Game with *complete information*

Initialize $X_{e,j}^k = 1,\ \forall k \in \mathcal{M}, e \in E, j \in \mathcal{C}$;
    $\Phi_0 = +\infty, n = 0, k = 0, m = 0$;
Extend edge e ($\forall e \in E$) to C parallel edges
(each extended edge is denoted by $(e, j),\ \forall j \in \mathcal{C}$);
repeat
    $n = n + 1, k = (k \mod M) + 1$;
    for each $e \in E, j \in \mathcal{C}$ **do**
        Update edge weight $W_{(e,j)}$ according to (3-8);
    end for
    Set $W_{(e,j)} = +\infty,\ \forall j \in \Gamma, e \in E$;
    Set $W_{(e,j)} = +\infty, \forall (e, j) \in \Omega$;
    Set $W_{(e,j)} = +\infty, \forall v \in \Lambda, e \in E(v), j \in \mathcal{C}$;
    Call *Dijsktra Algorithm* to find the shortest path for F_k in the **extended graph** with weight $W_{(e,j)}$ ($\forall e \in E, j \in \mathcal{C}$), and set (Source, Destination)$\leftarrow (S_k,\ D_k)$;
    Compute Φ_n according the shortest path;
    if $\Phi_n < \Phi_{n-1}$ **then**
        Update F_k's strategy according to the shortest path;
        $m = 0$;
    else
        $m = m + 1, \Phi_n = \Phi_{n-1}$;
    end if
until $m = M$
$s_k^* = \{(e, j)|X_{e,j}^k = 1, e \in E, j \in \mathcal{C}\}, \forall k \in \mathcal{M}$;
END.

then reducing $\Phi(s)$ equals to reducing

$$\Phi'(s) = \sum_{k' \in \mathcal{M}_k} \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^{k'} \omega_{k'} [\sum_{e' \in E} X_{e',j}^{k} \omega_k \theta_{e,e'}] + \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^k \zeta_{e,j}^k(s)$$

For the first term in $\Phi^{'}(s)$,

$$
\begin{aligned}
&\sum_{k^{'}\in\mathcal{M}_k}\sum_{e\in E}\sum_{j\in\mathcal{C}} X_{e,j}^{k^{'}}\omega_{k^{'}}[\sum_{e^{'}\in E} X_{e^{'},j}^{k}\omega_k\theta_{e,e^{'}}] \\
=&\sum_{e\in E}\sum_{j\in\mathcal{C}}\sum_{e^{'}\in E} X_{e^{'},j}^{k}\sum_{k^{'}\in\mathcal{M}_k} X_{e,j}^{k^{'}}\omega_{k^{'}}\omega_k\theta_{e,e^{'}} \\
=&\sum_{e\in E}\sum_{j\in\mathcal{C}} X_{e,j}^{k}\sum_{k^{'}\in\mathcal{M}_k}\sum_{e^{'}\in E} X_{e^{'},j}^{k^{'}}\omega_{k^{'}}\omega_k\theta_{e,e^{'}} \\
=&\sum_{(e,j)\in s_k}\sum_{k^{'}\in\mathcal{M}_k}\sum_{e^{'}\in E} X_{e^{'},j}^{k^{'}}\omega_{k^{'}}\omega_k\theta_{e,e^{'}}
\end{aligned}
$$

Note that we interchange the role of e and $e^{'}$ in the above equation and exploit $\theta_{e,e^{'}} = \theta_{e^{'},e}$. Hence, $\Phi^{'}(s)$ can be written as

$$
\begin{aligned}
\Phi^{'}(s) =& \sum_{(e,j)\in s_k}\sum_{k^{'}\in\mathcal{M}_k}\sum_{e^{'}\in E} X_{e^{'},j}^{k^{'}}\omega_{k^{'}}\omega_k\theta_{e,e^{'}} \\
&+\sum_{(e,j)\in s_k}\omega_k[\lambda_{e,j}^{k}(s)+2\varphi_{e,j}^{k}+2(1-A_{e,j})\gamma] \\
=&\sum_{(e,j)\in s_k}\sum_{k^{'}\in\mathcal{M}_k}\sum_{e^{'}\in E} X_{e^{'},j}^{k^{'}}2\omega_k\omega_{k^{'}}\theta_{e,e^{'}} \\
&+\sum_{(e,j)\in s_k}\omega_k[2\varphi_{e,j}^{k}+2(1-A_{e,j})\gamma].
\end{aligned}
$$

Therefore, if we set the weight of edge (e, j) in the extended graph when F_k is updating its strategy to be

$$
\begin{aligned}
W_{(e,j)} =& \sum_{k^{'}\in\mathcal{M}_k}\sum_{e^{'}\in E} X_{e^{'},j}^{k^{'}}2\omega_k\omega_{k^{'}}\theta_{e,e^{'}} \\
&+2\omega_k\varphi_{e,j}^{k}+2\omega_k(1-A_{e,j})\gamma,
\end{aligned}
$$

finding the shortest path in the extended graph will equal to reducing $\Phi^{'}(s)$ to the maximum extent (recall the optimality of Dijsktra Algorithm), which further reduces the potential function $\Phi(s)$ to the maximum extent. Besides, the route connectivity can be guaranteed by the property of Dijsktra Algorithm.

In terms of the time complexity, setting weight $W_{(e,j)}$ on the extended graph in each improvement step will consume $O(|E||\mathcal{C}|)$ and Dijsktra Algorithm is of $O(|V|^2)$. Hence the overall time complexity of each improvement step is $O(|E||\mathcal{C}| + |V|^2)$. □

Additionally, according to the proof to Theorem 2, $\Phi(s)$ only has one minimum point, so Algorithm 1 globally minimizes the potential function.

3.4 ϵ-Nash Equilibrium

Algorithm 1 offers us a method to compute the *exact* NE, where no players can reduce their own costs by unilaterally deviating the NE. Unfortunately, we cannot formally prove that Algorithm 1 will always reach the minimum of the potential function in polynomial time, even though simulation results show the fast-reaching tendency (see section 7). Alternatively, we can obtain an *approximate NE* or *ϵ-NE* in polynomial time, by modifying Algorithm 1. Firstly, we give the formal definition of the ϵ-Nash Equilibrium.

Definition 4 (ϵ-Nash Equilibrium): A strategy profile $s^* = (s_1^*, s_2^*, \cdots, s_M^*)$ is an ϵ-Nash Equilibrium if for any player F_k ($\forall k \in \mathcal{M}$) and its any strategy $s_k \subseteq S$,

$$TC_k(s_k^*; s_{-k}^*) \leq TC_k(s_k; s_{-k}^*) + \epsilon.$$

The above definition implies that no player can reduce its costs by ϵ if it unilaterally violates the ϵ-NE. Particularly, when $\epsilon = 0$, the ϵ-NE becomes the exact NE.

As a corollary of Property 3 of potential games, we have the following theorem.

Theorem 4: Under complete information, every Route-Switching Game has a unique ϵ-Nash Equilibrium.

To compute the pure ϵ-Nash Equilibrium, we only need to slightly modify Algorithm 1 by setting the condition in step 13 to be "$\Phi_n < \Phi_{n-1} - \epsilon$". By such a modification, we can conclude Theorem 5.

Theorem 5: The computation of the ϵ-Nash Equilibrium can terminate in $O(\frac{MP|E|^2}{\epsilon})$ steps, where $P = \min\{|\mathcal{C}|, M\}$.

Proof. From (2-2), it's obvious that

$$\lambda_{e,j}^{k}(s) = \sum_{k' \in \mathcal{M}_k} \sum_{e' \in E} X_{e',j}^{k'} \omega_{k'} \theta_{e,e'} \leq \max_{k' \in \mathcal{M}} \omega_{k'} \sum_{e' \in E} \sum_{k' \in \mathcal{M}} X_{e',j}^{k'}$$
$$\leq |E| \max_{k' \in \mathcal{M}} \omega_{k'}.$$

Define $U := \max_{k \in \mathcal{M}} \omega_k$, $Q := \max_{k \in \mathcal{M}, e \in E, j \in \mathcal{C}} \varphi_{e,j}^{k}$. Then according to (2-1) and (2-2), we have

$$\Phi(s) = \sum_{k \in \mathcal{M}} \sum_{(e,j) \in s_k} \omega_k [\lambda_{e,j}^{k}(s) + 2\varphi_{e,j}^{k} + 2(1 - A_{e,j})\gamma]$$
$$\leq U(|E|U + 2Q + 2\gamma) \sum_{e \in E} \sum_{j \in \mathcal{C}} \sum_{k \in \mathcal{M}} X_{e,j}^{k}$$
$$\leq P|E|U(|E|U + 2Q + 2\gamma),$$

where $P = \min\{|\mathcal{C}|, M\}$. According to the procedures of Algorithm 1, the value of the potential function will be reduced by at least ϵ after every M improvement steps otherwise the algorithm will stop. Hence, together with $\Phi(s) \geq 0$, we can conclude that the maximum number of improvement steps will be

$$\frac{M\Phi(s)}{\epsilon} \leq \frac{MP|E|U(|E|U + 2Q + 2\gamma)}{\epsilon}$$

Hence, the computation of an ϵ-Nash Equilibrium can terminate in $O(\frac{MP|E|^2}{\epsilon})$ steps. □

Chapter 4 Route-Switching Games with Incomplete Information

In the above chapters, we assume that all players have the exact information about others. However, obtaining exact parameters about other concurrent flows could be very difficult in the practical situations. As is often the case, we can obtain the statistical information about other flows, i.e., the incomplete-information scenario. In this chapter, we will extend our scheme to the incomplete-information game.

The proposed game with incomplete information can be indicated by the tuple $\mathcal{G} = \{G, A, \Gamma, S, TC, \mathbf{T}, \mathbf{p}\}$. The slight differences between this definition and that of the complete-information game lie in two aspects. Firstly, we introduce a type space $\mathbf{T} = \mathbf{T_1} \times \cdots \times \mathbf{T_M}$ to indicate the possible rates and packet sizes of data flows in the incomplete-information game, where $\mathbf{T_k}$ is the type space of data flow F_k. Then flow F_k's strategy s_k is a mapping from T_k to the strategy space S. Besides, the flow rate would be $r_k(t)$, the packet size would be $\mu_k(t)$, and the energy costs in channel j over link e would be $\varphi^k_{e,j}(t)$ if data flow F_k is of type t. Similarly, we define $\omega_k(t) = \frac{\mu_k(t)}{r_k(t)}$ and denote $T = \{t_1, \cdots, t_M\}$ the type profile, where t_k is the type of F_k. Secondly, each player only knows the type distribution $\mathbf{p}$ of other data flows over the type space $\mathbf{T}$, where $\mathbf{p} = (p(t_1, t_2, \cdots, t_M))_{T \in \mathbf{T}}$. Note that the Probability Density Function will be used when the type distribution is continuous. We assume the type distribution of each data flow is *independent*:

$$p(\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_M) = \prod_{k \in \mathcal{M}} p_k(\hat{t}_k),$$

where $p_k(\hat{t}_k)$ is the probability that data flow F_k is of type $\hat{t}_k$, shown by

$$p_k(\hat{t}_k) = \sum_{T \in \mathbf{T}: t_k = \hat{t}_k} p(t_1, t_2, \cdots, t_M).$$

Then we define the NE used in incomplete-information games, referred as the pure Bayesian Nash Equilibrium.

Definition 5 (Bayesian Nash Equilibrium): A strategy profile $s^* = (s_1^*, s_2^*, \cdots, s_M^*)$ is a pure Bayesian Nash Equilibrium if for any data flow F_k ($\forall k \in \mathcal{M}$) and its any type $t \in \mathbf{T_k}$, $s_k^*(t)$ satisfies:

$$s_k^*(t) = \arg \min_{s_k(t) \subseteq S} \mathbb{E}\{TC_k(s_k(t); s_{-k}^*(t_{-k}))|t_k = t\},$$

where $\mathbb{E}\{TC_k(s_k(t); s_{-k}^*(t_{-k}))|t_k = t\}$ is F_k's *expected cost function* when it is of type t and adopts $s_k(t)$.

Unlike Theorem 2 in the complete-information scenario, we won't directly offer a formal proof of the existence of the Bayesian NE. Instead, we'll first provide an algorithm to compute the Bayesian NE and then prove its correctness, shown in **Algorithm 2** and Theorem 6.

Algorithm 2 Computation of the Bayesian NE s^*

```
for each k = 1 : M do
    E{ω_k(t_k)} = Σ_{T∈T} p(t_1, t_2, ··· , t_M) ω_k(t_k);
    Compute E{φ^k_{e,j}(t_k)} (∀e ∈ E, j ∈ C) similarly;
end for
Compute the Nash Equilibrium s̄* using Algorithm 1 by replacing ω_k and φ^k_{e,j} with
    E{ω_k(t_k)} and E{φ^k_{e,j}(t_k)} (∀k ∈ M, e ∈ E, j ∈ C), respectively;
Set s*_k(t) = s̄*_k (∀k ∈ M, t ∈ T_k);
END.
```

Theorem 6: Algorithm 2 can compute a pure Bayesian Nash Equilibrium of the Routing-Switching Game with incomplete information.

Proof. We consider the contradiction and assume there exists a data flow F_k (of type t) whose strategy obtained by Algorithm 2 (i.e., $s_k^*(t) = \bar{s}_k^*$) is not its best response at the Bayesian NE. Then according to the definition of the Bayesian NE, F_k can change its strategy to $\bar{s}_k^{'}$ ($\bar{s}_k^{'} \neq \bar{s}_k^*$) so that

$$\begin{aligned} &\mathbb{E}\{TC_k(\bar{s}_k^{'}; s_{-k}^*(t_{-k}))|t_k = t\} \\ <&\mathbb{E}\{TC_k(\bar{s}_k^*; s_{-k}^*(t_{-k}))|t_k = t\}, \end{aligned} \tag{4-1}$$

where t_{-k} is the type profile excluding F_k's type t_k. F_k's *expected cost function* when it is of type t under $s_k(t)$ is

$$\begin{aligned} &\mathbb{E}\{TC_k(s_k(t);s_{-k}(t_{-k}))|t_k=t\} \\ =&\sum_{(e,j)\in s_k(t)}[\mathbb{E}\{\lambda^k_{e,j}(s_{-k}(t_{-k}))\}+\varphi^k_{e,j}(t)+(1-A_{e,j})\gamma]. \end{aligned} \tag{4-2}$$

Note that equation (4-2) is established on the fact that $\mathbb{E}\{\lambda^k_{e,j}(s_k(t);s_{-k}(t_{-k}))|t_k = t\} = \mathbb{E}\{\lambda^k_{e,j}(s_{-k}(t_{-k}))\}$ since the type distribution of each data flow is mutually independent and $\lambda^k_{e,j}(s)$ only depends on others' strategies $s_{-k}(t_{-k})$. Taking (4-2) to (4-1), we derive

$$\begin{aligned} &\sum_{(e,j)\in\bar{s}'_k}[\mathbb{E}\{\lambda^k_{e,j}(s^*_{-k}(t_{-k}))\}+\varphi^k_{e,j}(t)+(1-A_{e,j})\gamma] \\ <&\sum_{(e,j)\in\bar{s}^*_k}[\mathbb{E}\{\lambda^k_{e,j}(s^*_{-k}(t_{-k}))\}+\varphi^k_{e,j}(t)+(1-A_{e,j})\gamma]. \end{aligned} \tag{4-3}$$

Step 4 in *Algorithm 2* corresponds with a new complete-information game. To avoid the confusion of denotations, we will denote $\bar{s}$ an arbitrary strategy profile in the new complete-information game and the corresponding 0-1 strategy indication is $\bar{X}^i_{e,j}$ ($\forall i\in\mathcal{M}, e\in E, j\in\mathcal{C}$). By comparison, the corresponding strategy profile in the incomplete-information game is s which is a mapping from the type space to the strategy space, and the 0-1 indication is $X^i_{e,j}(t)$ ($\forall i\in\mathcal{M}, e\in E, j\in\mathcal{C}, t\in T_i$).

$$TC_k(\bar{s})=\sum_{(e,j)\in\bar{s}_k}[\hat{\lambda}^k_{e,j}(\bar{s})+\varphi^k_{e,j}(t)+(1-A_{e,j})\gamma].$$

Here,

$$\hat{\lambda}^k_{e,j}(\bar{s})=\beta\sum_{e'\in E}\sum_{k'\in\mathcal{M}_k}\bar{X}^{k'}_{e',j}\hat{\omega}_{k'}\theta_{e,e'},$$

$$\hat{\omega}_{k'}=\mathbb{E}\{\omega_{k'}(t_{k'})\}=\sum_{T\in\mathbf{T}}p(t_1,t_2,\cdots,t_M)\frac{\mu_{k'}(t_{k'})}{r_{k'}(t_{k'})}.$$

The above equation is actually derived from step 2 of Algorithm 2. Besides, according to step 6, the correspondence $\bar{s}_{k'}=s_{k'}(t)$ holds in the new complete-information game, for every $k' \in \mathcal{M}_k$ and $t \in T_{k'}$. Hence, $\hat{\lambda}^k_{e,j}(\bar{s})$ can be equally written as

$$\begin{aligned}
&\sum_{e' \in E}\sum_{k' \in \mathcal{M}_k} \bar{X}^{k'}_{e',j}\theta_{e,e'}\beta \sum_{T \in \mathbf{T}} p(t_1, t_2, \cdots, t_M)\frac{\mu_{k'}(t_{k'})}{r_{k'}(t_{k'})} \\
=&\sum_{T \in \mathbf{T}} p(t_1, t_2, \cdots, t_M)\beta \sum_{e' \in E}\sum_{k' \in \mathcal{M}_k} X^{k'}_{e',j}(t_{k'})\theta_{e,e'}\omega_{k'}(t_{k'}) \\
=&\mathbb{E}\{\lambda^k_{e,j}(s_{-k}(t_{-k}))\}.
\end{aligned}$$

This means that in the new complete-information game,

$$TC_k(\bar{s}) = \sum_{(e,j) \in \bar{s}_k} [\mathbb{E}\{\lambda^k_{e,j}(s_{-k}(t_{-k}))\} + \varphi^k_{e,j}(t) + (1 - A_{e,j})\gamma].$$

Taking the above equation to (4-3) and noticing that $s^*_{-k}(t_{-k}) = \bar{s}^*_{-k}$ for every $t_{-k} \in T_{-k}$, we can conclude that in the new complete-information game formed in step 5 of Algorithm 2,

$$TC_k(\bar{s}^{'}_k; \bar{s}^*_{-k}) < TC_k(\bar{s}^*_k; \bar{s}^*_{-k}),$$

which means that $\bar{s}^*$ is not the NE for the corresponding complete-information game, contradicting to the correctness of Algorithm 1. Hence *Theorem 6* has been proved. □

It should be mentioned that when the Bayesian ϵ-Nash Equilibrium is calculated, similar modifications (see section 3.4) should be made to Algorithm 1.

Chapter 5 Price of Anarchy

In this chapter, we will compare the performance of the proposed game with the socially optimal results obtained in centralized schemes. As for the complete-information game, we will compare the social costs and analyze the *Price of Anarchy* (PoA) ([16], Christodoulou and Koutsoupias, 2005: 1-7.). In terms of the incomplete-information scenario, the expected social costs as well as the Bayesian Price of Anarchy (BPoA) will be discussed.

5.1 Complete-Information Game

In the route-switching game with complete information, the metric of our interests is social costs, defined as the following.

Definition 6 (Social Costs): The social costs are the sum of all player's costs, i.e.,

$$SoC(s) = \sum_{k\in\mathcal{M}} TC_k(s).$$

Then we introduce the definition of Price of Anarchy in the complete-information game.

Definition 7 (Price of Anarchy): The price of anarchy is the ratio of social costs between the worst NE and the optimality in centralized schemes, i.e.,

$$PoA = \frac{SoC(s^*)}{\min_s SoC(s)},$$

where s^* is the worst NE point of the proposed game. Since the Route-Switching Game only has one unique NE, s^* is exactly the equilibrium obtained through Algorithm 1.

The following theorem shows that the PoA of the proposed game has an upper bound.

Theorem 7: The upper bound of the Price of Anarchy in the proposed game is ρ, where $\rho = \frac{2\max_{k\in\mathcal{M}}\omega_k}{\min_{k\in\mathcal{M}}\omega_k}$.

Proof. Let s^* denote the Nash Equilibrium obtained through Algorithm 1 and q be the strategy profile which can minimize the social costs. At the same time, we define $Z_1 := \max_{k\in\mathcal{M}} \omega_k$ and $Z_2 := \min_{k\in\mathcal{M}} \omega_k$. Thus $\rho = \frac{2Z_1}{Z_2}$. We can rewrite the expression of the potential function by

$$\begin{aligned}\Phi(s) =& 2\sum_{k\in\mathcal{M}} \omega_k[DC_k(s) + EC_k(s) + SC_k(s)] - \sum_{k\in\mathcal{M}} \omega_k DC_k(s)\\ \leq& 2Z_1 SoC(s) - \sum_{k\in\mathcal{M}} \omega_k DC_k(s)\end{aligned}$$

Similarly, we have

$$\Phi(s) \geq Z_2 SoC(s) + \sum_{k\in\mathcal{M}} \omega_k[EC_k(s) + SC_k(s)].$$

Since $\Phi(s^*)$ reaches the global minimum, we have

$$\begin{aligned}&Z_2 SoC(s^*) + \sum_{k\in\mathcal{M}} \omega_k[EC_k(s^*) + SC_k(s^*)] \leq \Phi(s^*)\\ &\leq \Phi(q) \leq 2Z_1 SoC(q) - \sum_{k\in\mathcal{M}} \omega_k DC_k(q).\end{aligned}$$

For the simplicity of denotations, we define

$$\alpha := \frac{\sum_{k\in\mathcal{M}} \omega_k[DC_k(q) + EC_k(s^*) + SC_k(s^*)]}{SoC(q)}$$

Then we can derive that

$$Z_2 SoC(s^*) \leq SoC(q)(2Z_1 - \alpha)$$

From the above inequality, we finally have

$$PoA = \frac{SoC(s^*)}{SoC(q)} \leq \frac{2Z_1}{Z_2} - \frac{\alpha}{Z_2} \leq \rho$$

□

Theorem 7 implies that the social costs under the NE derived from Algorithm 1 won't exceed ρ times of the minimum social costs even in the worst case. Here, ρ characterizes the heterogeneity of incoming data flows, which reflects the variance of flows' required time for transmitting one packet. In reality, such variance is not significant considering the transmission efficiency and costs ([10], Rappaport, 2002: 235.). Specially, when flows are homogeneous (ω_k is identical, $\forall k \in \mathcal{M}$), the NE yields less than twice of the minimum social costs ($\rho = 2$). Besides, ρ is a relatively loose bound, which means that the real PoA could be much less than ρ. The above two remarks of ρ imply that the obtained NE is actually close to the optimality in the practical situations (the PoA is usually below 1.5, as is shown in the simulation).

5.2 Incomplete-Information Game

As for the incomplete-information scenario, the corresponding concept is referred as the Bayesian Price of Anarchy (BPoA), which is defined as the following.

Definition 8 (Bayesian Price of Anarchy): The Bayesian price of anarchy is the ratio of *expected* social costs between the worst Bayesian NE and the optimal results obtained by centralized schemes, i.e.,

$$BPoA = \frac{\mathbb{E}\{SoC(s^*)\}}{\min_s \mathbb{E}\{SoC(s)\}},$$

Similarly, s^* corresponds to the Bayesian NE obtained through Algorithm 2.

The upper bound of the Bayesian Price of Anarchy is given in Theorem 8.

Theorem 8: The upper bound of the Bayesian Price of Anarchy in the proposed game is ϱ, where $\varrho = \frac{2\max_{k\in\mathcal{M}} \mathbb{E}\{\omega_k(t_k)\}}{\min_{k\in\mathcal{M}} \mathbb{E}\{\omega_k(t_k)\}}$.

Proof. Step 5 of Algorithm 2 corresponds with a new complete-information game. Similar to the denotations used in Appendix D, to avoid the confusion of denotations, we will denote $\bar{s}$ an arbitrary strategy profile in the new complete-information game and the corresponding 0-1 strategy indication is $\bar{X}^k_{e,j}$ ($\forall k \in \mathcal{M}, e \in E, j \in \mathcal{C}$). The corresponding strategy profile in the incomplete-information game is s which is a mapping from the type space to the strategy space, and the 0-1 indication is $X^k_{e,j}(t)$ ($\forall k \in \mathcal{M}, e \in E, j \in \mathcal{C}, t \in T_k$).

In the new complete-information game, the potential function under the strategy profile $\bar{s}$ is shown by

$$\Phi(\bar{s}) = \sum_{k\in\mathcal{M}} \mathbb{E}\{\omega_k(t_k)\}[DC_k(\bar{s}) + 2EC_k(\bar{s}) + 2SC_k(\bar{s})], \tag{5-1}$$

where

$$\mathbb{E}\{\omega_k(t_k)\} = \sum_{T\in\mathbf{T}} p(t_1,\cdots,t_M)\omega_k(t_k) \tag{5-2}$$

is derived from step 2 of Algorithm 2. Besides, the expressions of $DC_k(\bar{s})$, $EC_k(\bar{s})$ and $SC_k(\bar{s})$ in (5-1) are shown as the following.

$$DC_k(\bar{s}) = \sum_{(e,j)\in\bar{s}_k}\sum_{e'\in E}\sum_{k'\in\mathcal{M}} \bar{X}^{k'}_{e',j}\mathbb{E}\{\omega_{k'}(t_{k'})\}\theta_{e,e'}. \tag{5-3}$$

Similar to the discussion in Appendix D, we notice that the correspondence $\bar{s}_k = s_k(t)$ holds in the new complete-information game, for every $k \in \mathcal{M}$ and $t \in T_k$. Hence, taking (5-2) into (5-3), we can rewrite $DC_k(\bar{s})$ as

$$\begin{aligned} &DC_k(\bar{s}) \\ &= \sum_{(e,j)\in\bar{s}_k}\sum_{e'\in E}\sum_{k'\in\mathcal{M}} \bar{X}^{k'}_{e',j}\sum_{T\in\mathbf{T}} p(t_1,\cdots,t_M)\omega_{k'}(t_{k'})\theta_{e,e'} \\ &= \sum_{T\in\mathbf{T}} p(t_1,\cdots,t_M)\sum_{(e,j)\in s_k(t_k)}\sum_{e'\in E}\sum_{k'\in\mathcal{M}} X^{k'}_{e',j}(t_{k'})\omega_{k'}(t_{k'})\theta_{e,e'} \\ &= \mathbb{E}\{DC_k(s)\}. \end{aligned} \tag{5-4}$$

Then we transform $EC_k(\bar{s})$ in a similar way:

$$\begin{aligned} EC_k(\bar{s}) &= \sum_{(e,j)\in\bar{s}_k} \mathbb{E}\{\varphi^k_{e,j}(t_k)\} \\ &= \sum_{(e,j)\in\bar{s}_k}\sum_{T\in\mathbf{T}} p(t_1,\cdots,t_M)\varphi^k_{e,j}(t_k) \\ &= \sum_{T\in\mathbf{T}} p(t_1,\cdots,t_M)\sum_{(e,j)\in s_k(t_k)} \varphi^k_{e,j}(t_k) \\ &= \mathbb{E}\{EC_k(s)\}. \end{aligned} \tag{5-5}$$

Finally, we transform $SC_k(\bar{s})$ by

$$\begin{aligned} SC_k(\bar{s}) &= \sum_{(e,j)\in\bar{s}_k}(1-A_{e,j})\gamma \\ &= \sum_{(e,j)\in\bar{s}_k}\sum_{T\in\mathbf{T}}p(t_1,\cdots,t_M)(1-A_{e,j})\gamma \\ &= \sum_{T\in\mathbf{T}}p(t_1,\cdots,t_M)\sum_{(e,j)\in s_k(t_k)}(1-A_{e,j})\gamma \\ &= \mathbb{E}\{SC_k(s)\}. \end{aligned} \tag{5-6}$$

Here, we use the fact that

$$\sum_{T\in\mathbf{T}}p(t_1,\cdots,t_M)=1.$$

The above discussions imply that

$$\Phi(\bar{s})=\sum_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(s)+2EC_k(s)+2SC_k(s)\}.$$

Now we suppose that s^* is the Bayesian NE obtained through Algorithm 2 and q is the optimal strategy profile which can minimize the expected social costs. Define $Z_3=\max_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}$ and $Z_4=\min_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}$, and thus $\varrho=\frac{2Z_3}{Z_4}$. Then we have

$$\begin{aligned} \Phi(\bar{s}) =& 2\sum_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(s)+EC_k(s)+SC_k(s)\} \\ &-\sum_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(s)\} \\ \le& 2Z_3\mathbb{E}\{SoC(s)\}-\sum_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(s)\}. \end{aligned}$$

Here, $\mathbb{E}\{SoC(s)\})$ is the expected social costs:

$$\mathbb{E}\{SoC(s)\}=\sum_{k\in\mathcal{M}}\mathbb{E}\{DC_k(s)+EC_k(s)+SC_k(s)\}.$$

Similarly, we have

$$\Phi(\bar{s})\ge Z_4\mathbb{E}\{SoC(s)\}+\sum_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{EC_k(s)+SC_k(s)\}.$$

According to Algorithm 1, $\Phi(\bar{s}^*)$ reaches the gloabal minimum, i.e., $\sum_{k\in\mathcal{M}}\mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(s^*)+$

$2EC_k(s^*) + 2SC_k(s^*)\}$ reaches the global minimum, then we have

$$\begin{aligned}
& Z_4\mathbb{E}\{SoC(s^*)\} + \sum_{k\in\mathcal{M}} \mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{EC_k(s^*) + SC_k(s^*)\} \\
\leq & \sum_{k\in\mathcal{M}} \mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(s^*) + 2EC_k(s^*) + 2SC_k(s^*)\} \\
\leq & \sum_{k\in\mathcal{M}} \mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(q) + 2EC_k(q) + 2SC_k(q)\} \\
\leq & 2Z_3\mathbb{E}\{SoC(q)\} - \sum_{k\in\mathcal{M}} \mathbb{E}\{\omega_k(t_k)\}\mathbb{E}\{DC_k(q)\}.
\end{aligned}$$

The above inequality further implies that

$$Z_4\mathbb{E}\{SoC(s^*)\} \leq 2Z_3\mathbb{E}\{SoC(q)\}.$$

Finally, we derive the BPoA:

$$BPoA = \frac{\mathbb{E}\{SoC(s^*)\}}{\mathbb{E}\{SoC(q)\}} \leq \frac{2Z_3}{Z_4} = \varrho.$$

□

The above theorem implies that the BPoA is also related to the heterogeneity of incoming data flows. Similar to the discussion of the PoA, the real BPoA is not significant in the practical systems.

Chapter 6 Achieving the Social Optimality

Although Theorems 7 and 8 indicate that the inefficiency caused by players' selfishness is bounded by deterministic factors, it is more desirable that the proposed game can yield the minimum social costs so that $PoA = 1$ and $BPoA = 1$. To achieve this goal, we introduce an additional charging scheme to influence players' strategies and make them converge to the socially optimal results, which resembles the methods exploited in the works of Wu *et al.* ([20], Wu et al., 2011: 1-9.), ([21], Wu et al., 2008: 1-9.).

Specifically, as in many recent literatures (e.g., ([22], Wang et al., 2006: 2.), ([23], Liang et al., 2012: 3.)), we assume that there is a virtual currency in the system. When data flows are transmitted in the secondary network, the intermediate SU relays will charge each flow some additional virtual money, which may be interpreted as the compensation for SU relays, considering that they may also pay a certain amount of virtual money for leasing PUs' spectra. Note that such an additional charge is virtual, which does not correspond to any *real costs* (e.g., energy consumption, routing delay, etc.). Therefore, different from the routing costs and switching costs mentioned above, it should not be included in the social costs which measure the real system costs. In other words, our goal is to utilize the virtual charging scheme to minimize the system costs that exist in reality.

Next, we will first give the design of the additional charge and then explain its philosophy. Denoted $\pi_{e,j}^k$ the charge F_k has to pay for using channel j over link e. Then the expression of $\pi_{e,j}^k$ is given by

$$\pi_{e,j}^k = \sum_{e' \in E} \sum_{k' \in \mathcal{M}_k} X_{e',j}^{k'} (\alpha\omega_k - \omega_{k'})\theta_{e,e'} + (\frac{\alpha}{2} - 1)(\varphi_{e,j}^k + \gamma), \tag{6-1}$$

where α is a pre-determined constant and $\alpha \geq \max\{\frac{\rho}{2}, 2\}$ (ρ is the price of anarchy mentioned in Theorem 7) holds so that $\pi_{e,j}^k$ is non-negative. Under this design, the total virtual charge that flow F_k should pay on its two-dimensional route is

$$VC_k = \sum_{e \in E} \sum_{j \in \mathcal{C}} X_{e,j}^k \pi_{e,j}^k. \tag{6-2}$$

Hence, by summing the above virtual charge and the real costs (i.e., delay costs, energy costs and switching costs), we derive the *total loss* incurred on F_k's two-dimensional route under the strategy profile s (which is also denoted as $TC_k(s)$ for simplicity):

$$\begin{aligned} TC_k(s) =& DC_k(s) + EC_k(s) + SC_k(s) + VC_k(s) \\ =& \sum_{(e,j) \in s_k} \alpha \chi_{e,j}^k(s) + \frac{\alpha}{2}\Big(EC_k(s) + SC_k(s)\Big), \end{aligned} \tag{6-3}$$

where we define

$$\chi_{e,j}^k(s) := \sum_{e' \in E} \sum_{k' \in \mathcal{M}_k} X_{e',j}^{k'} \omega_k \theta_{e,e'}.$$

The philosophy behind the above design is to reshape the potential function to an ideal form and further ensure the NE of the proposed game can achieve the minimum social costs, which is shown in the following lemma and Theorem 9, respectively.

Lemma 1: If the above virtual charge is adopted, the potential function is reshaped to

$$\begin{aligned} \Phi(s) &= \alpha \sum_{k \in \mathcal{M}} [DC_k(s) + EC_k(s) + SC_k(s)] \\ &= \alpha SoC(s). \end{aligned} \tag{6-4}$$

To avoid lengthy presentation, we won't provide the formal proof to Lemma 1 in this thesis since it can be proved in a similar way to Theorem 1.

Recalling the discussion in section 3.2 that the minimum of the potential function corresponds to a NE and the fact that all players globally minimize the potential function through the implementation of Algorithm 1, we can conclude that the social costs under the NE will also reach the minimum if the potential function is proportional to $SoC(s)$. Hence, (6-4) is an ideal form of the potential function and Theorem 9 holds.

Theorem 9: If the above virtual charge is adopted, the Nash equilibrium s^* obtained through our scheme achieves the minimum social costs.

It should be mentioned that the expression of $W_{(e,j)}$ in step 6 of Algorithm 1 (i.e., equation (3-8)) should be modified when the virtual charge is incorporated.

$$W_{(e,j)} = \sum_{k' \in \mathcal{M}_k} \sum_{e' \in E} X^{k'}_{e',j}(\omega_k + \omega_{k'})\theta_{e,e'} + \varphi^k_{e,j} + (1 - A_{e,j})\gamma.$$

The proof to the correctness of the above equation is similar to that of Theorem 3, which is omitted here for brevity,

Additionally, Lemma 1 and Theorem 9 apply to the incomplete-information scenari-o as well, which indicates that the above virtual charging scheme can also ensure the minimum *expected social costs* under the Bayesian NE obtained throught Algorithm 2. To avoid the repeated explanation, we won't formally discuss the relevant results in the incomplete-information game here.

Chapter 7 Simulation

7.1 Simulation Settings

In this thesis, we exploit MATLAB as our simulation tool. For the network topology generation, we adopt the classical B-A algorithm to generate a (random) scale-free network. We also randomly assign a distance for each pair of nodes in the generated network following the unform distribution, with the distribution interval $[1, 8]$m for the first-hop, $[8, 16]$m for the second-hop, etc. The interference range of each node is 20m. The number of radios equipped by each SU is a random integer following the uniform distribution between $[1, 4]$. The flow rate r_k and packet size μ_k are uniformly distributed in [400,800]kbps and [400,600]Bytes, respectively. Besides, the previous channel assignment $A_{e,j}$ $(\forall e \in E, j \in \mathcal{C})$ as well as each PU's current channel state is 0 or 1 with an equal probability 0.5. In the following, the number of total channels is fixed to be 10, and the results are scaled by constants for the convenience of demonstration. Each data point is the 50-time average.

7.2 Simulation Results

We first simulate the Finite Improvement Property of the Route-Switching Game, shown in Figure 7.1 ($|V| = 30$). Initially, each player randomly picks a two-dimensional route, which incurs a large value of the potential function. After each improvement step, the potential is gradually reduced and finally reaches the minimum (a very small value but not zero, which might not be obvious in the figure due to the numerical scale), where a pure NE is reached. It also shows that games with more players require more improvement steps but the minimum can still be fast reached.

Figure 7.2 shows the variation of total channel switching times in the CRN with M and $|V|$. From this figure, we can observe that channel switching times increase with M almost linearly since more players imply more congestion and they are more likely to execute channel switching to avoid mutual interference. Besides, larger network size

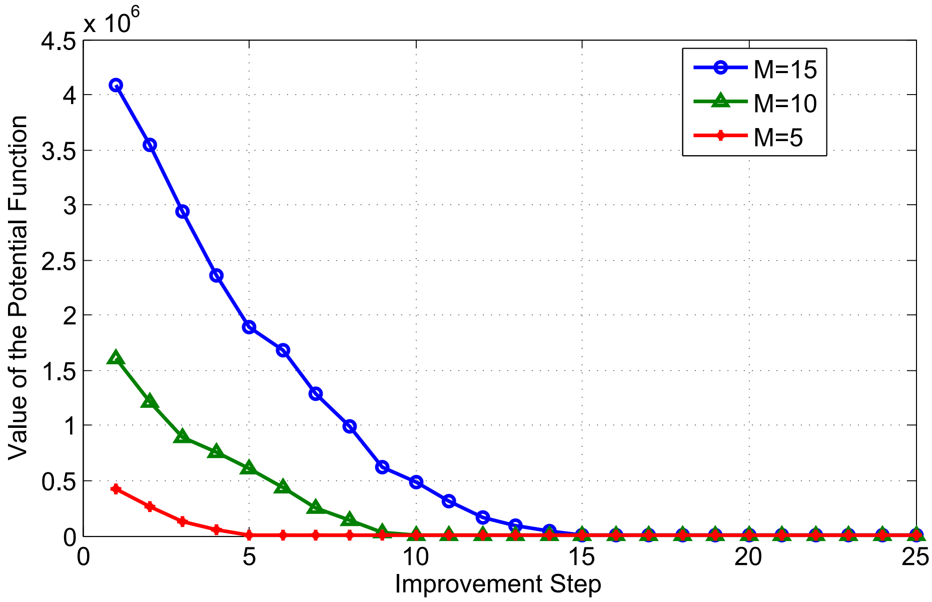

Figure 7.1 Finite Improvement Property of the proposed game

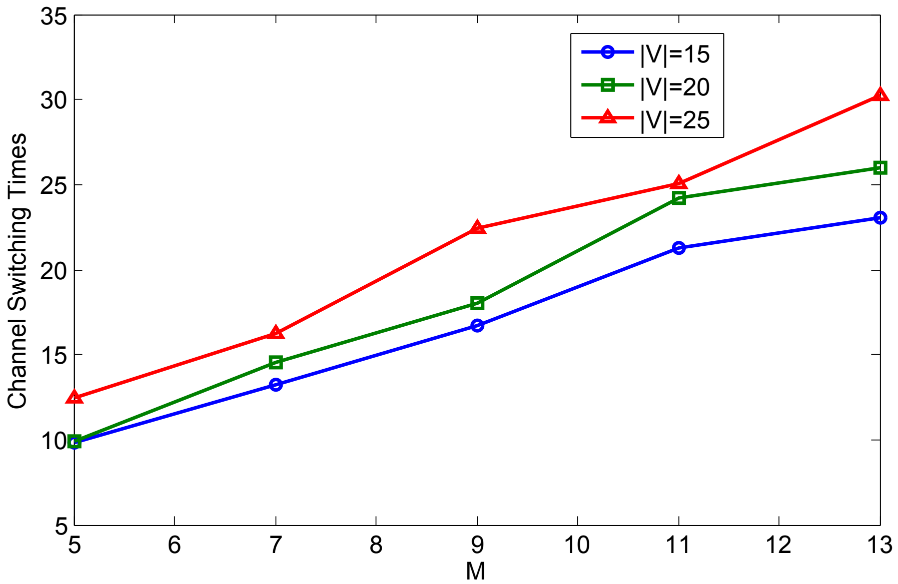

Figure 7.2 The variation of channel switching times with the number of players M and the network size $|V|$

also leads to higher switching frequency since the length of both frequency and spatial routes increases and the chances that players get influenced by spectrum dynamics are greatly raised.

We then focus on the performance of the $\epsilon-$NE, which sacrifices some precision in return for the time efficiency. Figure 7.3 shows the average number of improvement

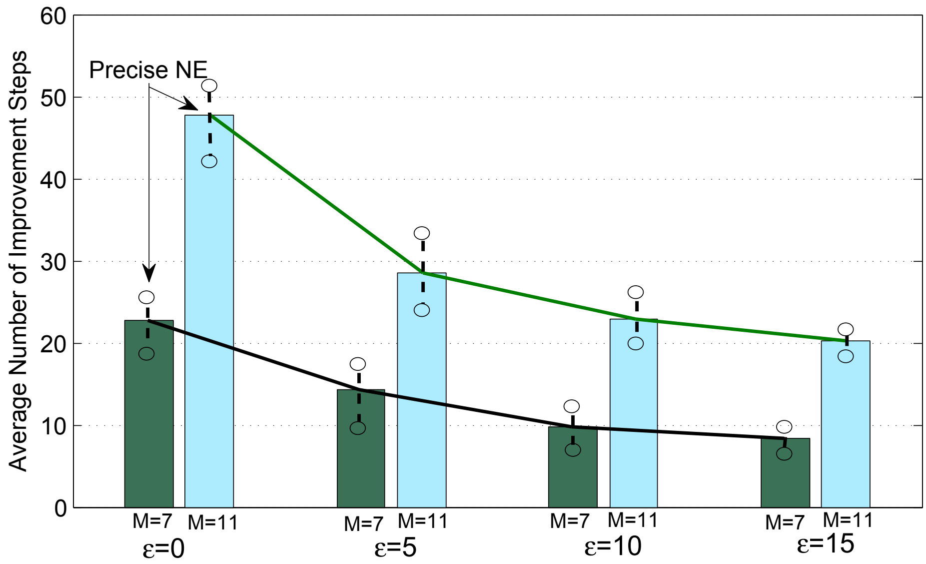

Figure 7.3 Average number of improvement steps used for finding the ϵ-NE (The dotted lines show the numerical range in 50 experiments.)

steps under different ϵ (note that $\epsilon = 0$ corresponds with the exact Nash Equilibrium), and $|V| = 20$. Dotted lines show the range in 50 experiments. We can apparently observe that the number of improvement steps reduces significantly with the increase of ϵ. Besides, we also compare the precision obtained with different ϵ in Figure 7.4, which reveals that the potential is raised almost linearly with the increase of ϵ, meaning that the precision of the NE drops. Therefore, a careful design of ϵ is required to achieve the balance between the time efficiency and the precision.

In Figure 7.5, we illustrate the comparison between the social optimality (obtained by exhaustive search) and the NE of the proposed game with and without the charging scheme, in terms of social costs ($|V| = 15$). When the charging scheme is not adopted, we can observe that the social costs under the NE are very close to the optimal results (the PoA is below 1.5) but the performance gap still exists. After the introduction of the virtual charge, there's an obvious efficiency improvement, and the PoA is exactly 1, which means that the NE obtained through our scheme achieves the socially optimal results.

Next, we illustrate the social costs incurred in our game with complete and incomplete information, respectively ($|V| = 20$), as is shown by the two blue curves in Figure

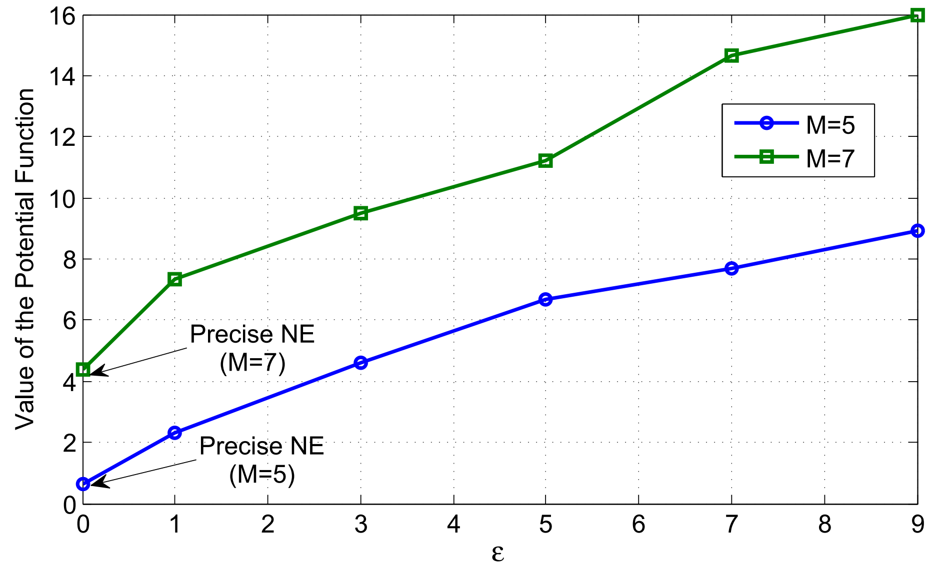

Figure 7.4 The potential value of the ϵ-Nash Equilibrium under different ϵ

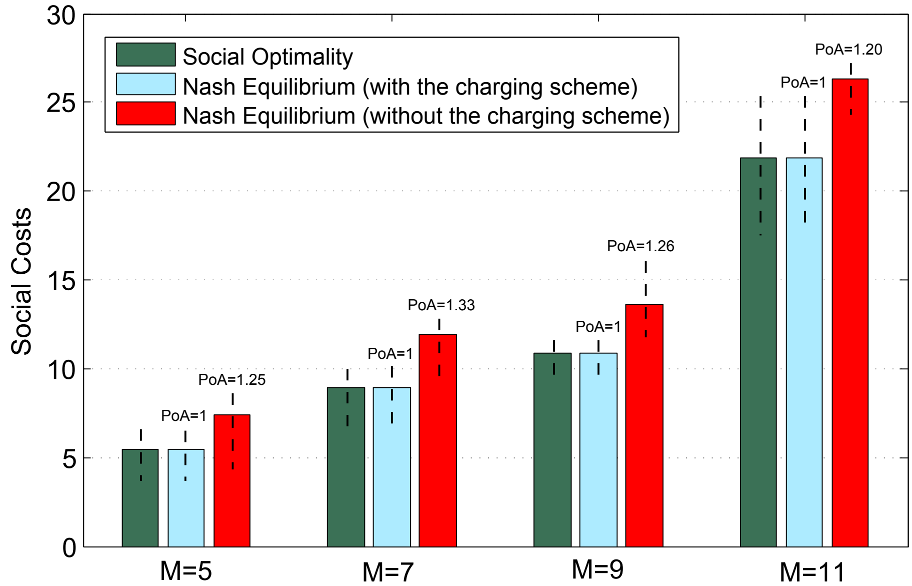

Figure 7.5 Social costs under the social optimality and the NE of the proposed game (with and without the charging scheme) (The dotted lines show the numerical range in 50 experiments.)

7.6. We can observe that the social costs obtained in the complete-information scenario are fewer than those in the incomplete-information game, which demonstrates the advantage of full knowledge. However, as is illustrated by the red curve in Figure 7.6, the performance gap (measured in the percentile form and in terms of the social costs) be-

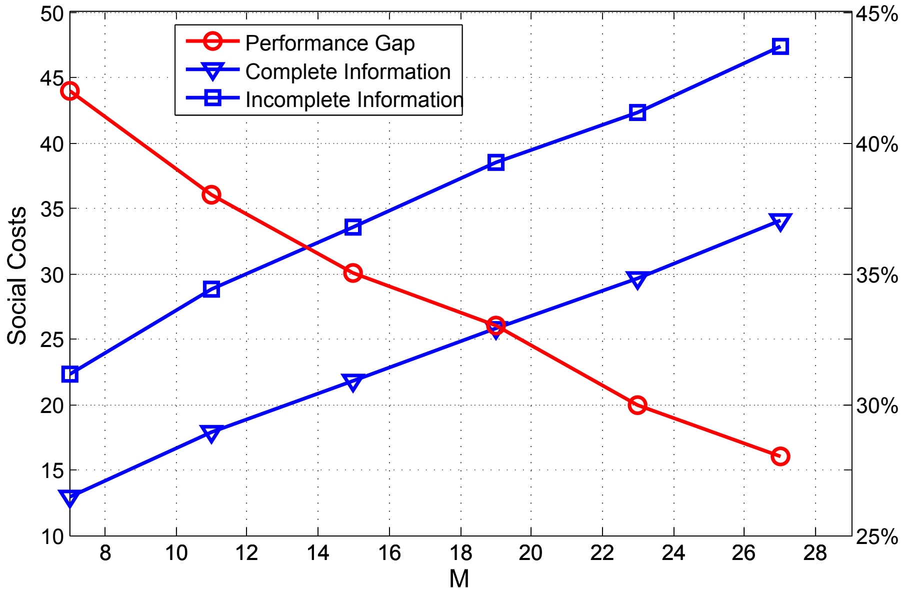

Figure 7.6 Blue curves illustrate the comparison of social costs between complete- and incomplete-information game. The red curve demonstrates the performance gap (measured in percentile) between the two scenarios.

tween the two scenario shrinks with the increase of the number of flows in the network. Thus, the information advantage is gradually obscure since players' *real* type distribution is closer to the probability distribution when more and more players participate in the game.

Chapter 8 Related Works

For the two-dimensional routing, there have been some literatures focused on the similar problem in conventional wireless networks. A joint channel assignment and routing protocol was investigated by Chiu *et al.* ([24], Chiu et al., 2009: 1-10.) for the IEEE 802.11-based mobile ad hoc networks. A novel routing metric was introduce by Wu *et al.* ([25], Wu et al., 2006: 1-12.) to achieve channel assignment and routing in multi-hop wireless networks. Kodialam *et al.* ([26], Kodialam and Nandagopal, 2005: 1-15.) and Alicherry *et al.* ([27], Alicherry et al., 2006: 1-12.) jointly considered the channel assignment and multi-flow scheduling in the mesh networks. Unfortunately, most of these existing works are neither robust enough to handle spectrum mobility in CRNs nor able to weigh the benefits and costs of route switching.

In the context of CRNs, spectrum dynamics have been heatedly studied recently. For example, Southwell *et al.* ([2], Southwell et al., 2012: 1-9.) proposed the spectrum mobility games in CRNs in order to derive a channel switching plan which minimizes the congestion level, and Liang *et al.* ([5], Liang et al., 2012: 1-6.) applied game-theoretical approaches to the spectrum selection problem in face of the channel dynamics. A robust channel assignment scheme in the multi-hop CRN was provided by Zhao *et al.* ([28], Zhao and Cao, 2012: 1-9.) to handle PUs' channel reclaiming behaviors. Besides, many market-driven methods were also proposed for the channel selection problem in CRNs. For example, Yang *et al.* ([29], Yang et al., 2011: 1-9.) proposed a pricing schemes to enable dynamic spectrum access control with random protocols. Gao *et al.* ([4], Gao et al., 2011: 1-13.) offered a novel contract-theoretical mechanism to dynamically allocate spectrum resource in secondary networks. Auction-based approaches ([3], Wang et al., 2010: 1-10.) were investigated by Wang *et al.* to achieve efficient spectrum trading under the dynamic channel availability. Liang *et al.* ([23], Liang et al., 2012: 1-11.) introduced the Random Leader as a dynamic central control entity which designs socially optimal plans for SUs, so as to enable dynamic channel assignment and spectrum handoff in face of spectrum mobility. In the spatial domain, Caleffi *et al.* ([30], Caleffi et al., 2012: 1-11.) considered the diversity effects of spatial routes and proposed an optimal routing metric for CRNs, and a connectivity-based routing scheme for the cognitive ad hoc networks was introduced in the work of Abbagnale *et al.* ([31], Abbagnale and Cuomo, 2010: 1-5.). However, these schemes only considered either the frequency or the spatial domain. There're still no major works focusing on two-dimensional routing or spectrum-mobility-incurred route switching in CRNs.

As for the potential games, the earliest concept was provided in the work of Mondere *et al.* ([15], Mondere and Shapley, 1996: 1-20.), which provided detailed analysis about the existence of Nash equilibria, the Finite Improvement Property, the ϵ-Nash equilibrium, etc. Further, potential games have been widely utilized as a relatively new game-theoretical model which can characterize a wide range of games, including the classical congestion game ([16], Christodoulou and Koutsoupias, 2005: 1-7.), ([32], L. M. Law and Liu, 2012: 1-9.). It has already demonstrated its importance through many successful applications to practical problems. For instance, Chen *et al.* ([17], Chen and Huang, 2012: 1-10.) built a potential-game-theoretical framework for spatial spectrum access and provided efficient methods to compute the Nash equilibrium. In the work of Nie *et al.* ([18], Nie and Comaniciu, 2005: 1-10.), potential games were used to solve the adaptive channel allocation problems in cognitive radio networks. Song ([19], Song et al., 2011: 1-12.) provided a potential game perspective about the optimal gateway selections in multi-domain wireless networks. Besides, analysis about the price of anarchy in potential games was introduced in many following works. For instance, Christodoulou *et al.* ([16], Christodoulou and Koutsoupias, 2005: 1-7.) provided the analysis about the price of anarchy in several representative congestion games (a special case of general potential games). Law *et al.* ([32], L. M. Law and Liu, 2012: 1-9.) systematically investigated the price of anarchy in general types of wireless congestion games.

The charging scheme we utilize in this thesis to achieve the social optimality resembles the methods exploited in the works of Wu *et al.* ([20], Wu et al., 2011: 1-9.), ([21], Wu et al., 2008: 1-9.). In their works, charging scheme is used in non-cooperative wireless network for coordinating players' behavior towards the socially optimal results of channel assignment.

Chapter 9 Conclusion

In this thesis, we investigate the spectrum-mobility-incurred route-switching problem in spatial and frequency domain for multi-hop CRNs. We formulate the proposed problem as the Route-Switching Game and prove that this game possesses a potential function. Then an iterative algorithm for finding the NE and a polynomial-time algorithm for computing the $\epsilon-$NE are provided in the thesis. The proposed game is further extended to the incomplete-information scenario and an algorithm for calculating the Bayesian NE is offered. Finally, we show that the price of anarchy has an upper bound, and a charging scheme is further introduced so that the NE obtained in our scheme can achieve the socially optimal results as centralized schemes do.

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